

Advances in Project Management Series¹

The Case for Project Net Present Value (NPV) and NPV Risk Models

By Martin Hopkinson

A project may be worth doing provided that its costs are more than justified by its benefits. This principle lies at the heart of any project business case. If we can forecast the financial value of a project's benefits, we can compare it to the project's cost to test how attractive the business case is. NPV modelling is a way of performing this test. Its approach to discounting cash whereby future cash can be compared to today's value is considered to make it a robust method for making financially-based project selection and approval decisions.

The NPV method

Today's value of an amount of cash is its present value. If we know the cost of tying up cash or delaying its receipt, we can calculate the present value of cash at different points in the future. The rate of cost can be expressed as a discount rate. For example, if the discount rate (D) is 10%, the annual cost of tying up £100 cash is £10. On this basis, we would need $£100(1+D) = £100(1+0.1) = £110$ one year in the future to compensate for having £100 today. This is equivalent to calculating that the present value of £110 one year in the future would be $£110/(1+D) = £100$. Thus, if an amount of value of cash in one year in the future is written as C_1 and its present value written as P_1 , then $P_1 = C_1 / (1+D)$. In general, since the associated costs accumulate at a compound rate, for a point in time that is n years in the future the present value P_n of an actual amount of cash C_n is calculated by the formula:

$$P_n = C_n / (1+D)^n.$$

The Net Present Value (NPV) method for project financial modelling is usually applied by calculating the present value of the cash flow for each annual period of the extended project life cycle. Costs produce a negative cash flow, whilst benefits contribute positively. The cash flow for each period can thus be calculated by deducting the cost forecast for that period from the cash value of the forecast benefits. Having calculated the present value for the net cash flow during each period the project NPV is calculated by summation. The formula for project NPV is thus:

$$NPV = \sum_{t=0}^n C_t / (1 + D)^n \quad \text{using year-end discount factors, or}$$

¹The *Advances in Project Management series* includes articles by authors of program and project management books published by Gower in the UK in April 2016. See the author's profile at the end of this article.

$$NPV = \sum_{t=0}^n C_t / (1 + D)^{n-0.5} \quad \text{using mid-year discount factors}$$

where:

C_t = the net cash flow over a period of time (typically 1 year),

t = the period of time during which that cash flow takes place,

D = the discount rate (rate of loss in the value of cash expressed as a percentage - typically per annum) and

n = the number of periods of time periods (typically years) over which NPV is calculated

The discount factor is the value by which a year's cash flow is multiplied by to obtain its present value. In practice, many projects are modelled using year-end discount factors. However, on projects in which costs and benefits materialise continuously throughout each year, the use of mid-year discount factors is usually more appropriate.

A Simple Example

Figure 1 is a simple example that illustrates how NPV can be modelled. It is based on a project that is forecast to cost £2.5m over a two-year period and achieve benefits of £3.5m over a period commencing 18 months into the project and ending at the end of Year 5. An undiscounted forecast of the project's net value would thus be £3.5m - £2.5m = £1m. Figure 1 shows how applying a discount rate of $D = 4\%$ affects this calculation. Note that the 4% rate used in the example above is comparatively low and reflects the use of constant cost forecasts for costs and benefits i.e. without the effects of inflation.

Base estimates		Costs and benefits broken down by year (£k)					
	Value (£k)	Year 1	Year 2	Year 3	Year 4	Year 5	Total
Delivery cost	2,500	1,250	1,250	0	0	0	2,500
Net benefits	3,500	0	500	1,000	1,000	1,000	3,500
NPV calculation		Year 1	Year 2	Year 3	Year 4	Year 5	Total
Cash flow: Benefits - Cost (£k)		-1,250	-750	1,000	1,000	1,000	1,000
Discount Rate	4%						
Mid-year discount factor		0.9806	0.9429	0.9066	0.8717	0.8382	
Cash flow Present Value (£k)		-1,226	-707	907	872	838	684
Cumulative NPV (£k)		-1,226	-1,933	-1,026	-155	684	

Figure 1: Illustration of the calculation of Project Net Present Value (NPV)

Figure 1 shows that the project's NPV is £684k. The reduction in value from the undiscounted calculation of £1m illustrates the effect of discounting. Discounting cash flows also usually

increases the project payback period. Thus, for example, whereas, in an undiscounted model the project would become cash positive as from the end of Year 4, Figure 1 shows that cumulative NPV would not reach this point until almost a quarter of the way through the fifth year.

Advantages of using the NPV Approach in Projects

NPV supports a rigorous approach to modelling the value of projects. Discount rates vary significantly from one organisation to another, the cost of capital being driven by the relevant markets' perception of risk. The NPV method thus allows the financial strength of a project business case to be assessed in the organisation's own financial context. It is also sensitive to the timing of cash flows, favouring projects with costs that materialise later or deliver benefits earlier than others.

A key property of present value calculations is that they align all forecasts to a scale based on today's value of cash. This enables a fair comparison between investments and/or projects regardless of their duration or phasing of cash flows, making it a useful portfolio management tool. It also provides a basis on which groups of independent projects can be aggregated. Thus, if we have two independent projects A and B:

$$\text{NPV (Project A + Project B)} = \text{NPV (Project A)} + \text{NPV (Project B)}$$

The same property also allows different options for the same project to be compared. Such options are often mutually exclusive since you would only choose one combination of options for any particular project. A key point is that such choices often concern the scope of a project and how it should be delivered and are made in its earliest project phases prior to full implementation. NPV models can thus be a valuable tool for developing a project solution that optimises the trade-off between costs and benefits as a project solution is being developed.

A final point in favour of NPV modelling is that it requires a project to take into account costs and benefits beyond just the period during which a project manager will be accountable for its delivery. This can help counter a danger with a traditional project management approach in which project managers may treat the phases for which they are accountable with greater importance than subsequent phases. For example, they might choose to control project implementation phase costs at the expense of the value that the project adds to operations. The NPV approach requires all relevant impacts of a project to be taken into account.

NPV Risk Models

NPV risk modelling is an extension to the NPV method that can provide an improved way of representing the implications of project-specific risk. Risk can be defined as being "the effect of uncertainty on objectives" (ISO 31000, 2009). The use of NPV modelling is consistent with the premise that a project's objective is to optimise value for money. Therefore, if value for

money is a key objective and we are able to extend the NPV technique to model the effect of uncertainty, we can produce an NPV risk model.

In common with a number of sources, this paper takes the word to uncertainty to mean “lack of certainty”. We can thus contrast risk models with deterministic NPV models that use single point input estimates; with the single points implying an assumption of certainty. As a result of recognising that both costs and benefits are uncertain, an NPV risk model will calculate the degree to which the overall project outcome may vary as a consequence.

Modern approaches to project risk analysis account for the potential implications of both negative and positive effects. However, it remains the case that the typical outcome of project cost risk analysis is the identification of a need for more funding than calculated from the use of base estimates. A similar issue affects the estimation of benefits. The realisation of benefits is also exposed to risk. But here, experience has demonstrated a tendency for benefits to be overestimated rather than underestimated. Downside risk is usually thus larger than upside risk.

Simple Example of a Net Present Value Risk Model

The following example is designed to illustrate how an NPV risk model is structured and how its results can be displayed. The project is the same as the previous example shown in Figure 1.

This example uses Monte Carlo simulation to operate the model. The model requires the use of a suitable tool (such as @RISK for Excel or ModelRisk) that operates as an add-on to a conventional spreadsheet tool. For the purposes of simplicity, it has been assumed that risk can be simulated appropriately by substituting the single values of overall project cost and benefits used in the previous example with triangular probability density functions. Table 1 summarises the input estimates that have been used and Figure 2 shows the associated probability density functions. From these we can see that it has been estimated that risk associated with the project cost implies that it could vary from as little as £2m (the optimistic extreme estimate) to as much as £3.5m (the pessimistic extreme estimate). The most likely cost relative to other possibilities (mode) has been estimated as being £2.7m. This compares to the previous cost estimate of £2.5m, the additional £0.2m being attributable to an assessment that the effect of cost risk is more likely to be adverse than not.

	Min	Mode	Max	Apportionment of cost and benefits
Project cost (£k)	2,000	2,700	3,500	Even spread over Years 1 and 2
Project benefits value (£k)	2,000	3,500	4,500	Even spread from middle of Year 2 to end of Year 5

Table 1: Inputs to the simple Net Present Value Risk model example

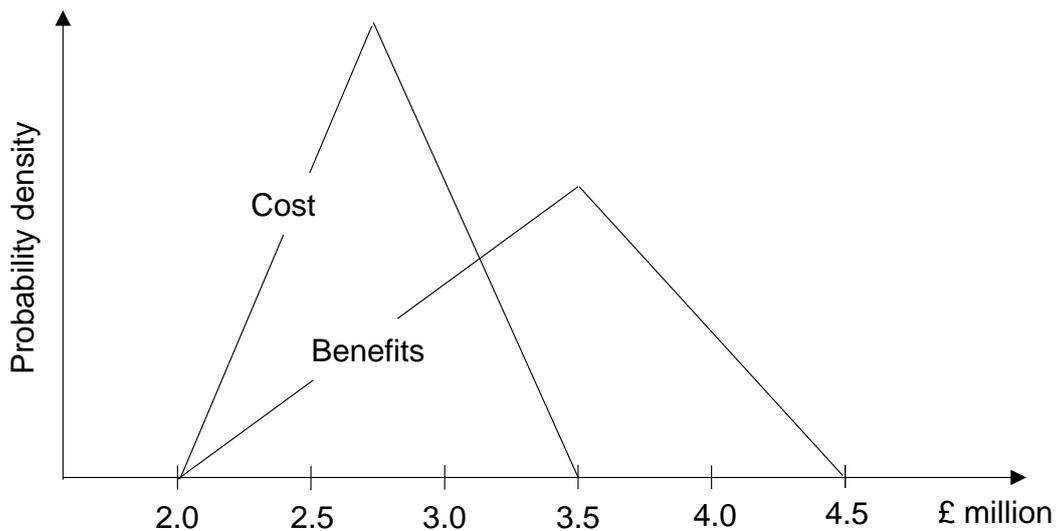


Figure 2: Illustration of the probability density functions used by the Example NPV risk model

When the model is run the Monte Carlo process simulates a large number of iterations. 5,000 iterations is a sufficient number for many models, although the speed of modern tools usually makes the use of more practicable. During each iteration, a random value selected from the cost probability function is used to represent project cost and an independent random value similarly selected for benefits. The corresponding project NPV is then calculated using this pair of values in the same manner as that used for the earlier NPV model example. The risk model thus uses the same NPV calculation structure as that shown in Figure 1. It also uses the same discount rate $D = 4\%$.

Although the probability density functions in Figure 2 will allow you to see that, for most iterations, net benefits will exceed costs, it should also allow you to realise that there will be some iterations in the Monte Carlo simulation for which the reverse is the case and that the calculated value of NPV will be negative. For example cost = £3,143k and benefits = £2,482k is a plausible combination for an iteration, even if the reverse combination is more likely. The results produced by the risk model are shown by the histogram and cumulative distribution graphs in Figure 3. They show that, although the project is more likely to have a positive NPV than not, negative NPV is a significant possibility.

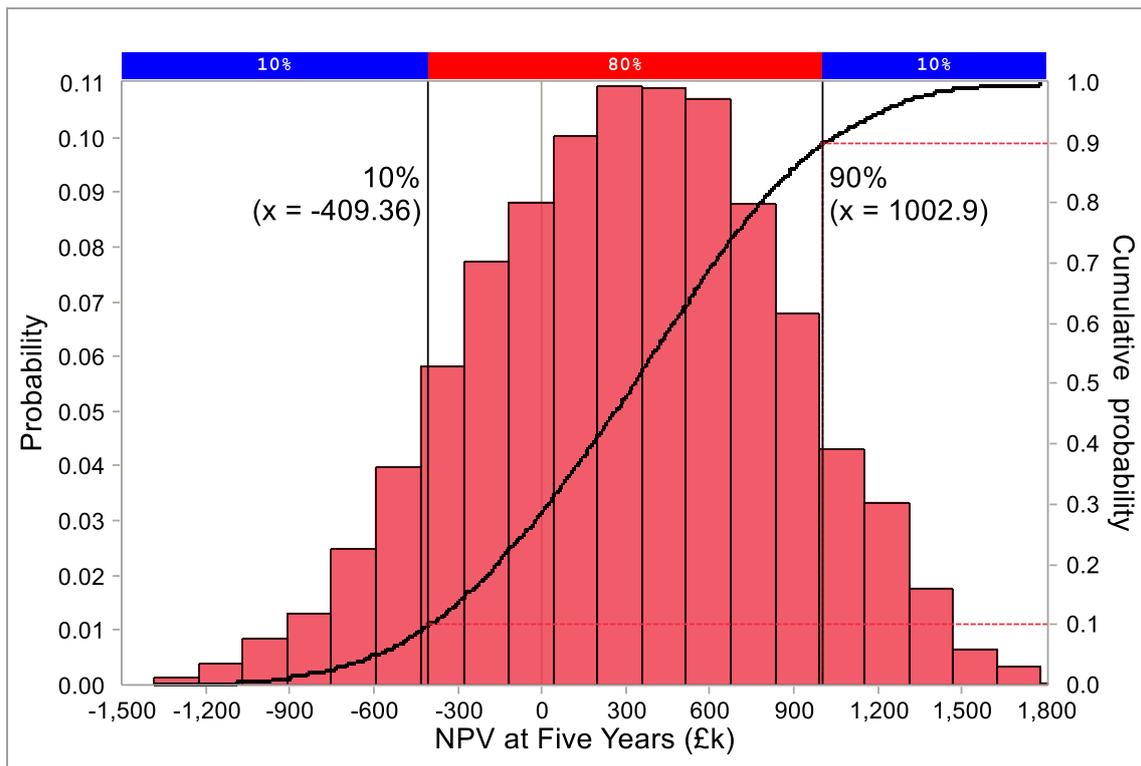


Figure 3: Results of the simple NPV Risk Model produced by ModelRisk

Risk modelling tools provide a variety of statistical results. In the case of this example, the results include:

- Mean NPV = £310K
- Median (P50) NPV = £330K
- NPV Standard deviation = £540K
- Skewness = -0.11

The mean NPV forecast of £310K can be compared to the £684K forecast by the earlier deterministic NPV model 2. The lower value produced by the risk model is due to the fact that the mean values of both input probability density functions are less optimistic than the values used for the NPV model.

Figure 3 illustrates three different methods for representing project risk using the results from NPV risk modelling. One method is to use a histogram. A second method is to use a cumulative probability curve. These curves are frequently referred to as s-curves owing to their characteristic shape. Both methods are capable of illustrating the full range of the results but have different advantages. The advantage of using histograms is that they illustrate asymmetry in the results more readily. An advantage of using cumulative probability curves are that, where several results are available for comparison purposes, it is easier to illustrate their implications by overlaying s-curves. Another advantage of cumulative probability curves are that they can be illustrated as being directly linked to percentile points, which are in themselves a third method for representing results.

The method of using modelling results percentile points is illustrated in Figure 3 by the 10th (P10) and 90th (P90) percentiles. The model forecasts an 80% probability that the project would achieve an NPV of a value within these boundaries. Percentile point data represents risk in two ways. First, the interval between percentiles indicates the risk associated with variance in outcome. Second, percentile points can be associated with specific objectives. For example, if the project objective was to deliver an NPV of zero or greater, we could deduce from Figure 3 that there would be a probability of project failure of approximately 28% i.e. the point on the cumulative probability scale that intersects the s-curve as it crosses the zero point on the x-axis.

Advantages of NPV Risk Modelling

One benefit of NPV risk modelling is the increased richness in the results that it produces. For example, Figure 3 illustrates the limitations of single value forecast produced by a deterministic model such as that shown in Figure 1. Including the effects of project-specific risk directly in NPV models also allows an organisation to differentiate between projects that are high risk and others that are not. This can be contrasted with the approach of organisations that make an allowance for the effects of project risk by applying a standard increase to the discount rate.

The application of a sound risk estimating process can also help to address sources of bias that may affect single point estimates of the type used in deterministic models. Moreover, an early use of risk estimates may be useful in countering the phenomenon of optimism bias in first-pass project forecasts based on immature data, thereby avoiding later problems caused by management expectations of costs and benefits having been set at an unrealistic level.

Finally, NPV risk modelling can be used to support the management of risk during the earliest phases of a project; the time during which uncertainty is greatest. Since the value of opportunities should exceed costs and tends to be realised during a later period, the risk to benefits often exceeds that of the risk to cost. Given that opportunities to manage risk to benefits are often best exploited as a project is being planned, NPV risk models can be used to develop a risk-robust project solution, thus integrating the project risk management and business case development processes. When I have used this approach in practice, the consequences have been beneficial to both business case development and the effectiveness of the risk management process itself.

Notes

1. The examples illustrated in this paper are taken from the book *Net Present Value and Risk Modelling for Projects* (Routledge, 2016)
 2. The two models have been made as simple as possible for the purposes of explaining the basic calculation process. Excel versions of these models, and other more complex models, can be downloaded from www.projectnpv.com.
 3. Care needs to be taken to align the discount rate with the treatment of inflation in the model. Some discount rates are calculated at nominal rates i.e. including inflation, whereas others are calculated in real terms. This and other NPV modelling-related issues are described in more detail in the above book and web site.
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About the Author



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Martin Hopkinson is the Director of Risk Management Capability Limited and has 30 years' experience as a project manager, project risk management specialist and consultant. His experience has been gained across a wide variety of industries and engineering disciplines and includes multi-billion pound projects and programmes.

Martin's first book, *The Project Risk Maturity Model*, concerns the risk management process. His contributions to Association for Project Management (APM) guides such as *Directing Change* and *Sponsoring Change* reflect his belief in the importance of project governance and business case development.

In his new book *Net Present Value and Risk Modelling for Projects* he brings these subjects together by showing how NPV and risk modelling techniques can be used to optimise projects and support project approval decisions. ([To learn more about the book, click here.](#))